The NJL Model for Quarks in Hadrons and Nuclei
- Part I: Quarks and Mesons -

W. Bentz (Tokai Univ., Japan)

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Introduction

Q1: What is the Nambu-Jona-Lasinio (NJL) model?
A1: A quark model based on relativistic field theory.
Characteristic: Contact interactions between quarks. Easy to handle, very successful to describe hadrons, nuclear matter and quark matter.
Q2: Who invented this model?
A2: Nambu and Jona-Lasinio in 1960, as a model for elementary nucleons. Re-discovered in the 1980th as a model for quarks.
Q3: What is this model good for?
A3: We can describe

- hadrons (nucleons, mesons) as bound states of quarks
- nuclear matter and nuclei in terms of quarks ($\Rightarrow$ Quark nuclear physics; Medium modifications)
- phases of strongly interacting matter at high densities ($\Rightarrow$ Neutron stars, supernova matter)

Motivations: Symmetries (1)

- **Success of constituent quark model.** Basic inputs are: Nonrelativistic quarks ($M_u \simeq M_d \simeq 300 - 400$ MeV), and symmetry of wave functions.
  But: Quarks of QCD are almost massless ($m \simeq 0$) and relativistic, and structure of wave functions should emerge from dynamics. $\Rightarrow$ Generate constituent quark masses and wave functions dynamically from interactions.
- The Lagrangian of any quark model should be symmetric under the global **gauge transformations**

$$\psi(x) \to e^{i\alpha}\psi(x)\,, \qquad \psi(x) \to e^{i\vec{\alpha}\cdot\vec{\tau}}\psi(x)$$

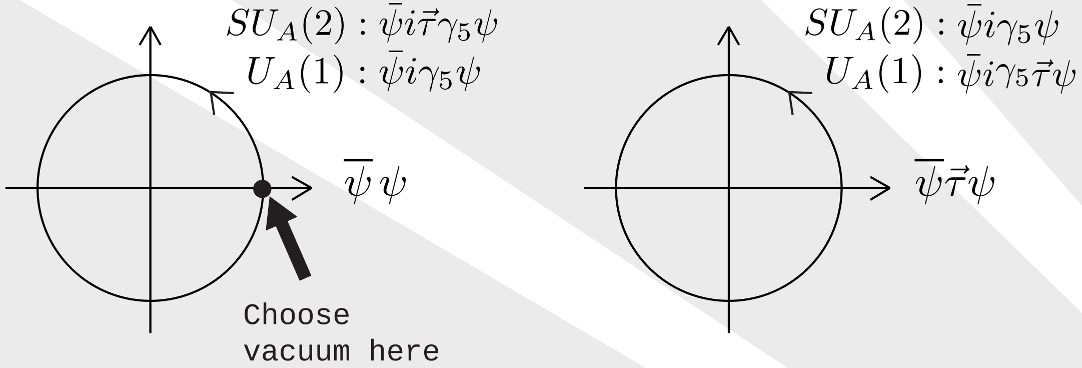
  where $\psi = (\psi_u, \psi_d)$ is the flavor SU(2) quark field. $\Rightarrow$ conserved currents $j^\mu = \overline{\psi}\gamma^\mu\psi$ and $\vec{j}^\mu = \overline{\psi}\gamma^\mu\vec{\tau}\psi$.
- The interaction Lagrangian should also be symmetric under the **chiral** $SU_A(2)$ **transformation**

$$\psi(x) \to e^{i\vec{\alpha}\cdot\vec{\tau}\gamma_5}\psi(x)$$

  If only the quark mass term $-m\overline{\psi}\psi$ breaks this symmetry, we are led to the PCAC relation: $\partial_\mu\left(\overline{\psi}\gamma^\mu\gamma_5\vec{\tau}\psi\right) = 2m\,\overline{\psi}i\gamma_5\vec{\tau}\psi$.

Motivations: Symmetries (2)

- This chiral symmetry should be **spontaneously broken** and the pion should emerge as a **Goldstone boson**.
The above chiral $SU_A(2)$ transformation can be expressed as a rotation in the plane of $\sigma = \overline{\psi}\psi$ and $\vec{\pi} = \overline{\psi} i\gamma_5 \vec{\tau}\psi$:



If the energy of the system along a circle is lower than at the origin ($\sigma = \pi = 0$), we may choose one of the states on the circle as the "**vacuum**". (In the figure: $\sigma \neq 0$, $\pi = 0$.) A small chiral rotation (moving up along the circle) leads to another (degenerate) vacuum, which differs from the original one by the appearance of a π field $\Rightarrow$ π is a massless "Goldstone boson".

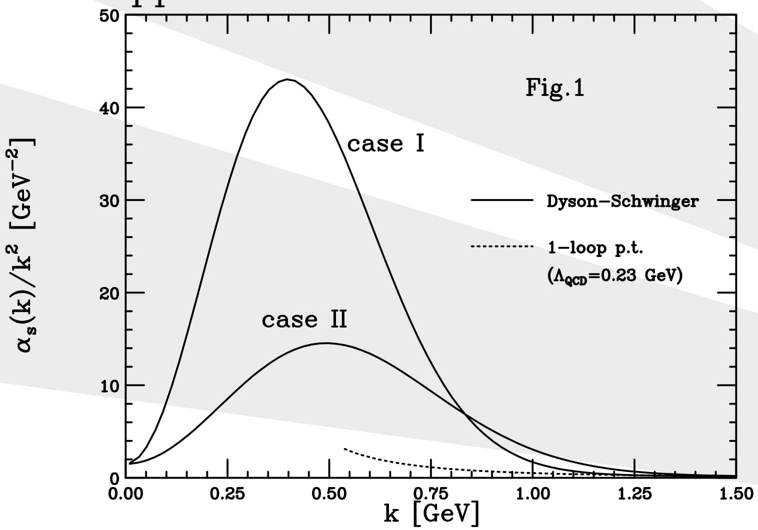
- The chiral $U_A(1)$ symmetry $\psi \to \exp(i\alpha\gamma_5)\psi$ is unwanted (no isoscalar Goldstone boson is observed!), and should be broken explicitly by the interaction.

Motivations: Interaction (1)

- How to model the elementary **qq interaction**? By meson exchange, like the nuclear force?
  But: Mesons are also composite particles!
  $\Rightarrow$ Meson exchange between quarks should be the *result*, but not the *input* of the model.
- QCD based Dyson-Schwinger theories indicate: qq interaction looks like gluon exchange, but with a modified "**running coupling**" $\alpha_s(k)$:

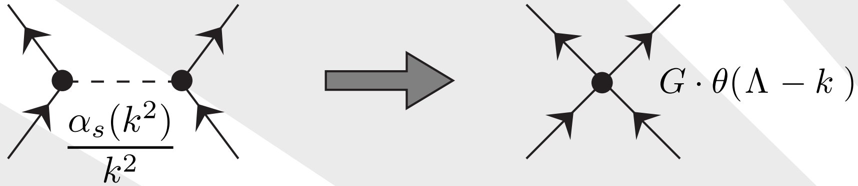
$$V(k) = \frac{\lambda^a}{2}\gamma_\mu \left(\frac{\alpha_s(k)}{k^2}\right)\gamma^\mu \frac{\lambda^a}{2}$$

qq interaction at low momenta



Interaction and Lagrangian

Interaction is very strong at small k: **Infrared enhancement**.
$\Rightarrow$ For low momenta $(k < \Lambda \simeq$ 1 GeV) we may approximate

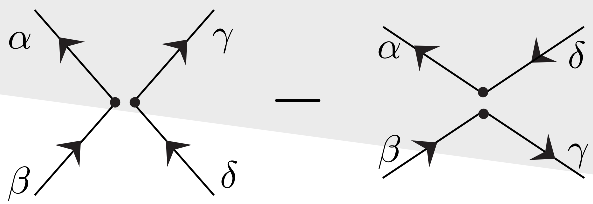


where G is a 4-Fermi coupling constant. This looks like a contact interaction, but restricted to low momenta!

Using the flavor SU(2) quark field $\psi = (\psi_u, \psi_d)$, we can write the corresponding **Lagrangian density** as

$$\mathcal{L} = \overline{\psi}\,(i\,\not\nabla - m)\,\psi - G\left(\overline{\psi}\frac{\lambda^a}{2}\gamma_\mu\psi\right)^2$$

From Wick's theorem: There are 2 diagrams for the interaction between a quark and an antiquark: (time runs from left to right!)



Fierz transformations

If we use **Fierz transformations** (s. Notes!) to rewrite the interaction identically, we can save work and calculate only the first ("direct") diagram!

$$\begin{aligned}\mathcal{L}_I &= \frac{1}{2}\left(\mathcal{L}_I + \mathcal{L}_{I,\mathrm{Fierz}}\right) = G_\pi \left[\left(\overline{\psi}\psi\right)^2 + \left(\overline{\psi} i\gamma_5 \vec{\tau}\psi\right)^2\right] \\ &+ \text{other } q\overline{q} \text{ channels}\end{aligned}$$

where $G_\pi = \frac{2}{9}G$. This is the **most familiar form of the NJL model**, since it shows the chiral symmetric interactions in the scalar (σ) and pseudoscalar (π) $q\overline{q}$ channels, which are most important.

An example for other $q\overline{q}$ channels is the interaction in the vector meson (ω) channel: $-G_\omega \left(\overline{\psi}\gamma^\mu\psi\right)^2$, where $G_\omega = \frac{1}{9}G$.
$U_A(1)$ symmetry breaking is described by another 4-Fermi interaction - the "determinant interaction". Its effect can be incorporated into a redefinition of the constants G_π, G_ω.

Mean field approximation

Use the **mean field (Hartree) approximation** to define the constituent quark mass M as an effect of the quark self energy: Adding $\left(-M\overline{\psi}\psi + \mathrm{const}\right)$ and subtracting again, we get:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{res}}$$

where (writing only the scalar and pseudoscalar interaction terms)

$$\begin{aligned} \mathcal{L}_0 &= \overline{\psi}\left(i \not\nabla - M\right)\psi + \mathrm{const} \\ \mathcal{L}_{\mathrm{res}} &= (M-m)\overline{\psi}\psi + G_\pi \left[\left(\overline{\psi}\psi\right)^2 + \left(\overline{\psi} i \gamma_5 \vec{\tau}\psi\right)^2\right] - \mathrm{const} \end{aligned}$$

Now assume that there is a nonzero expectation value of $\overline{\psi}\psi$ in the vacuum ("**quark condensate**"):

$$\overline{\psi}\psi = \langle\overline{\psi}\psi\rangle + :\overline{\psi}\psi:$$

where the second term is the normal ordered product. Then determine M and const by the requirements that $\mathcal{L}_{\mathrm{res}}$ has **no c-number term** and **no linear term** $\propto :\overline{\psi}\psi:$ (i.e., $\mathcal{L}_{\mathrm{res}}$ is a "true" residual 4-Fermi interaction).

Gap equation

These requirements give

- the **gap equation**:

$$
\begin{aligned}
M &= m - 2G_\pi \langle \overline{\psi}\psi \rangle = m + 2iG_\pi \lim_{\tau \to 0^+} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr} S_F(k) e^{ik_0\tau} \\
&= m + 48i\, G_\pi M \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M^2 + i\epsilon}
\end{aligned}
$$

  ($S(k)$ is the Feynman propagator of a quark with mass M.)
  After regularization of the integral, this has to be solved for M.

- the constant term:

$$
\mathrm{const} = -\frac{(M-m)^2}{4G_\pi}
$$

We **finally** get: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{res}}$ with

$$
\begin{aligned}
\mathcal{L}_0 &= \overline{\psi}\,(i\,\not{\nabla} - M)\,\psi - \frac{(M-m)^2}{4G_\pi} \\
\mathcal{L}_{\mathrm{res}} &= G_\pi \left[(:\overline{\psi}\psi:)^2 + (:\overline{\psi} i\gamma_5 \vec{\tau}\psi:)^2 \right] + \text{other channels}
\end{aligned}
$$

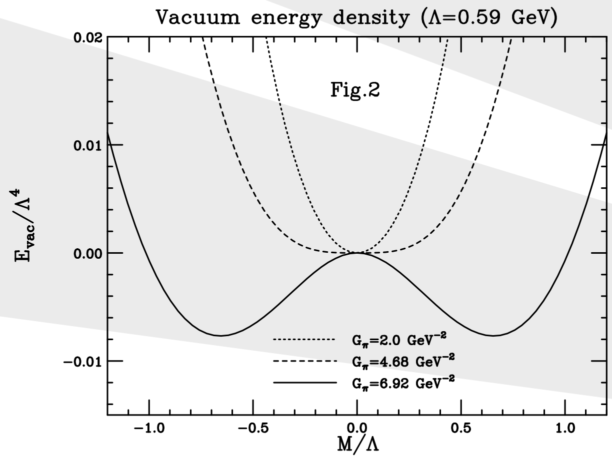
Spontaneous breaking of chiral symmetry (1)

For the case $m = 0$, the gap equation has 2 solutions: (i) **trivial solution** $M = 0$, (ii) **nontrivial solution** satisfying

$$1 = 48i\, G_\pi \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 - M^2 + i\epsilon}$$

Which is the "correct" solution? Compare the vacuum energy densities $\mathcal{E}$ ("effective potentials") for these 2 cases: From $\mathcal{L}_0$,

$$\mathcal{E}_{\mathrm{vac}}(M) - \mathcal{E}_{\mathrm{vac}}(M=0) = -12 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left(\sqrt{M^2 + k^2} - k \right) + \frac{M^2}{4G_\pi}$$

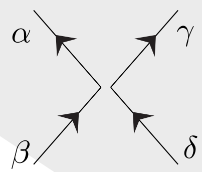


Spontaneous breaking of chiral symmetry (2)

If G_π is larger than some critical value, the energy on the chiral circle $\sigma^2 + \vec{\pi}^2 = M^2/4G_\pi^2$ is lower than for $\sigma = \vec{\pi} = 0$. The choice $\vec{\pi} = 0$ in the vacuum corresponds to spontaneous breaking of the chiral symmetry, and the pion becomes a Goldstone boson (which will be verified later).

BS equation for mesons (1)

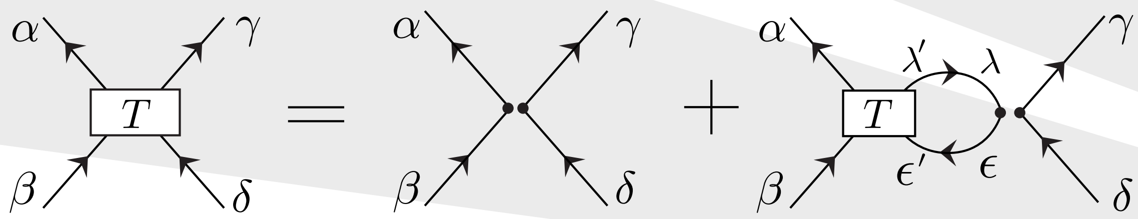
From $\mathcal{L}_{\mathrm{res}}$, we have the **Feynman rule** for the $q\bar{q}$ interaction in the scalar and pseudoscalar channels (time runs from left to right):



$$2iG_\pi \left[(1)_{\gamma\delta}(1)_{\alpha\beta} - (\gamma_5\vec{\tau})_{\gamma\delta}(\gamma_5\vec{\tau})_{\alpha\beta}\right]$$

Then the equation for the $q\bar{q}$ scattering matrix (**Bethe- Salpeter equation**) becomes for fixed total 4-momentum p^μ:

$$T_{\gamma\delta,\alpha\beta}(p) = K_{\gamma\delta,\alpha\beta} + \int \frac{\mathrm{d}^4k}{(2\pi)^4} K_{\gamma\delta,\epsilon\lambda} S_{\epsilon'\epsilon}(k) S_{\lambda\lambda'}(p+k) T_{\lambda'\epsilon',\alpha\beta}(p)$$



BS equation for mesons (2)

Inserting the form $K_{\gamma\delta,\alpha\beta} = C\,\Gamma_{\gamma\delta}\Gamma_{\alpha\beta}$, where C is a constant and Γ a matrix, and assuming the solution of the form

$$T_{\gamma\delta,\alpha\beta}(p) = t(p)\,\Gamma_{\gamma\delta}\Gamma_{\alpha\beta}$$

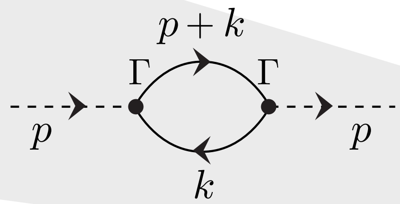
we get for the scalar function $t(p)$ the simple equation:

$$t(p) = C - iC\,\Pi(p^2)\,t(p) \;\Rightarrow\; t(p) = \frac{C}{1 + iC\Pi(p^2)}$$

with the "**bubble graph**" (polarization propagator)

$$\Pi(p^2) \equiv i \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\Gamma S(p+k)\Gamma S(k)\right]$$

$t(p)$ **has a pole** at $p^2 = \mu^2$ if $1 + iC\Pi(\mu^2) = 0$.
σ channel $\Rightarrow \Gamma = 1,\; C = 2iG_\pi$; π channel $\Rightarrow \Gamma = \gamma_5\tau,\; C = -2iG_\pi$.

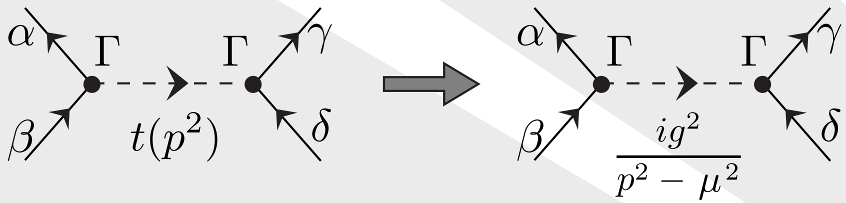
$\pi(p^2) = -i$ 

Bound state masses and vertex functions

Expanding $\Pi(p^2)$ near the pole as
$\Pi(p^2) = \Pi(\mu^2) + (p^2 - \mu^2)\Pi'(\mu^2) + \ldots$, we see that near the pole

$$t(p) \to \frac{ig^2}{p^2 - \mu^2}$$

where $g^2 \equiv (-1/\Pi'(\mu^2))$.

$T_{\gamma\delta,\alpha\beta} =$ 

This looks like the exchange of an elementary meson! Therefore, it is natural to interpret μ as the **meson mass** and g as the **quark-meson coupling constant**.
For the **case of pion** ($\Gamma = \gamma_5 \tau$): By comparing the pion pole condition $1 + 2G_\pi \Pi_\pi(m_\pi^2) = 0$ to the gap equation for $m = 0$ (exact chiral symmetry), it is easy to see that $m_\pi^2 = 0 \Rightarrow$ **Pion is really the Goldstone boson**.

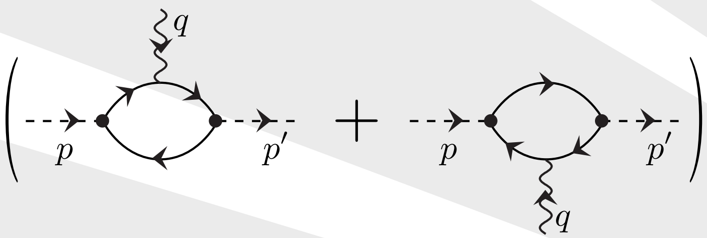
Application 1: Charge form factor of π^+ (1)

Definition of **electromagnetic current of pion**:

$$\frac{1}{\sqrt{4E_pE_{p'}}}\int \mathrm{d}^4z\, e^{-iq\cdot z}\langle \mathbf{p}'|\overline{\psi}(z)\gamma^\mu\left(\frac{1}{6}+\frac{\tau_3}{2}\right)\psi(z)|\mathbf{p}\rangle \equiv (2\pi)^4\,\delta^{(4)}(p'-p-q)j^\mu(q)$$

Here we use covariant normalization of states:
$\langle \mathbf{p}'|\mathbf{p}\rangle = 2(2\pi)^3E_p\delta^{(3)}(\mathbf{p}'-\mathbf{p})$, where $E_p = \sqrt{\mathbf{p}^2+m_\pi^2}$.
According to Mandelstam's theory of bound state matrix elements, the current $j^\mu(q)$ can be calculated from **Feynman diagrams**:

$$j^\mu(q) = \frac{1}{\sqrt{4E_{p'}E_p}}$$



The π^+ charge form factor is then defined by

$$j^\mu(q) \equiv \frac{(p'+p)^\mu}{\sqrt{4E_pE_{p'}}}F_\pi(Q^2) \quad (Q^2 \equiv -q^2 > 0 \text{ for electron scattering})$$

Charge form factor of π^+ (2)

Inserting $\gamma_5 \tau_+ g$ at the left pion-quark vertex and $\gamma_5 \tau_- g$ at the right vertex, we obtain

$$j^\mu(q) = \frac{1}{\sqrt{4E_p E_{p'}}} 6ig^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{Tr}_D \left[\gamma_5 S(p'+k)\gamma^\mu S(p+k)\gamma_5 S(k)\right]$$

This can be evaluated by using one of the regularization schemes (see Notes!).

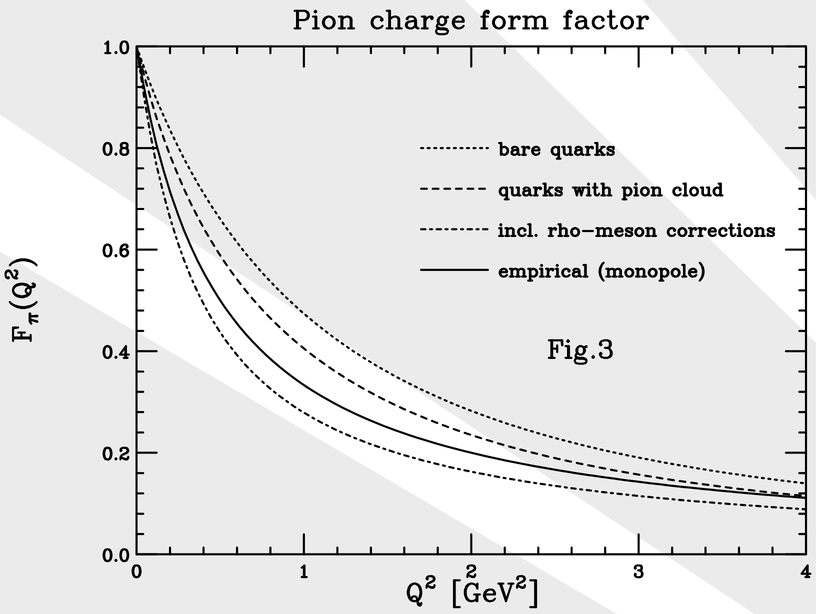
Check current conservation and charge conservation: By using elementary Ward-like identities

$$\begin{aligned} q_\mu \left(S(k')\gamma^\mu S(k)\right) &= -\left(S(k') - S(k)\right) \\ S(k)\gamma^\mu S(k) &= -\frac{\partial S(k)}{\partial k_\mu} \end{aligned}$$

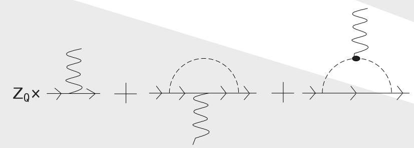
we get

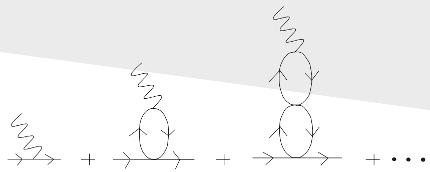
$$\begin{aligned} q_\mu j^\mu &= \frac{-g^2}{\sqrt{4E_p E_{p'}}} g^2 \left(\Pi_\pi(p'^2) - \Pi_\pi(p^2)\right) = 0 \quad (\text{because } p'^2 = p^2 = m_\pi^2) \\ j^\mu(q=0) &= \frac{-g^2}{2E_p}\left(\frac{\partial \Pi_\pi(p^2)}{\partial p_\mu}\right) = \frac{p^\mu}{E_p} \quad (\text{from} \quad \text{definition of } g^2) \end{aligned}$$

Results for pion charge form factor



"bare quarks" (dotted line) refers to the formula on previous slide using the proper-time cut-off, and "monopole" (solid line) is the empirical pion form factor determined from experiment:
$F_{\pi,\mathrm{emp}} = 1/(1 + Q^2/(0.5\,\mathrm{GeV}^2))$.
The following corrections due to intrinsic quark form factors ($\gamma^\mu \to \gamma^\mu F_q(Q^2)$) are also shown:
(i) pion cloud around quarks, and (ii) $\gamma - \rho$ coupling (cf. Vector Meson Dominance model).





Application 2: Quark distributions (1):

If we set $q = 0$ in the formula for the current, and replace the quark charge operator by the number operator for up quarks $(1 + \tau_3)/2$, we get a "**number sum rule**" for the up quark:

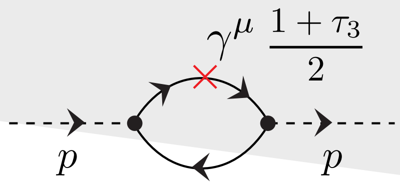
$$\frac{1}{2E_p}\langle \mathbf{p}|\overline{\psi}(0)\gamma^\mu \frac{1+\tau_3}{2}\psi(0)|\mathbf{p}\rangle = N_u \frac{p^\mu}{E_p}$$

where $N_u = 1$ is the number of u-quarks in π^+.
If we define the **up-quark correlation function in the pion** as

$$M^\mu(p,k) = i\int \mathrm{d}^4\omega\, e^{ik\cdot\omega}\,\langle \mathbf{p}|\overline{\psi}(0)\gamma^\mu \frac{1+\tau_3}{2}\psi(\omega)|\mathbf{p}\rangle$$

we can write the above number sum rule in the form

$$-\,i\int \frac{\mathrm{d}^4k}{(2\pi)^4} M^\mu(p,k) = 2p^\mu N_u$$

$N_u = \dfrac{1}{2p^\mu}\Bigg($ 

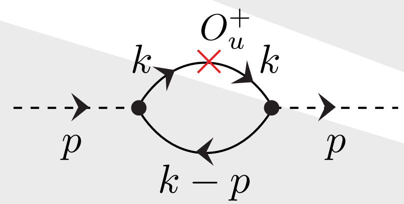
$\Bigg)$ $(\mu$ fixed$)$

Quark momentum distribution in π^+ (2):

We see: The operator insertion $\gamma^\mu(1+\tau_3)/2$ counts the number of u-quarks with all possible momenta $\Rightarrow$ The operator insertion $\mathcal{O}_u^\mu \equiv \gamma^\mu(1+\tau_3)/2 \cdot \delta(x - k^\mu/p^\mu)$ will count the number of u-quarks which have a **fraction** x **of the momentum component** p^μ.
In the description of **Deep Inelastic Scattering** (DIS), one needs the case $\mu = +$. (Then $k^+ \equiv (k^0 + k^3)/\sqrt{2}$ is the "light-cone plus-component" of k^μ.) We then get for the number of u-quarks with fraction x of the pion momentum component p^+:

$$f_u^{\pi^+}(x) = \frac{-i}{2p^+}\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\delta(x - \frac{k^+}{p^+})M^+(p,k)$$

with normalization $\int_0^1 f_u^{\pi^+}(x)\mathrm{d}x = N_u = 1$.

$$f_u(x) = \frac{1}{2p^+}\Bigg($$  $$\Bigg)$$

Quark momentum distribution in π^+ (3):

The distribution $f_u^{\pi^+}(x)$ is obtained from the above Feynman diagram as:

$$f_u^{\pi^+}(x) = \frac{ig^2}{2p^+}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma_5\tau_- S(p+k)\gamma^+\frac{1+\tau_3}{2}S(p+k)\gamma_5\tau_+ S(k)\right]\delta(x-\frac{k^+}{p^+})$$

We can perform the k^- integral by residues, using

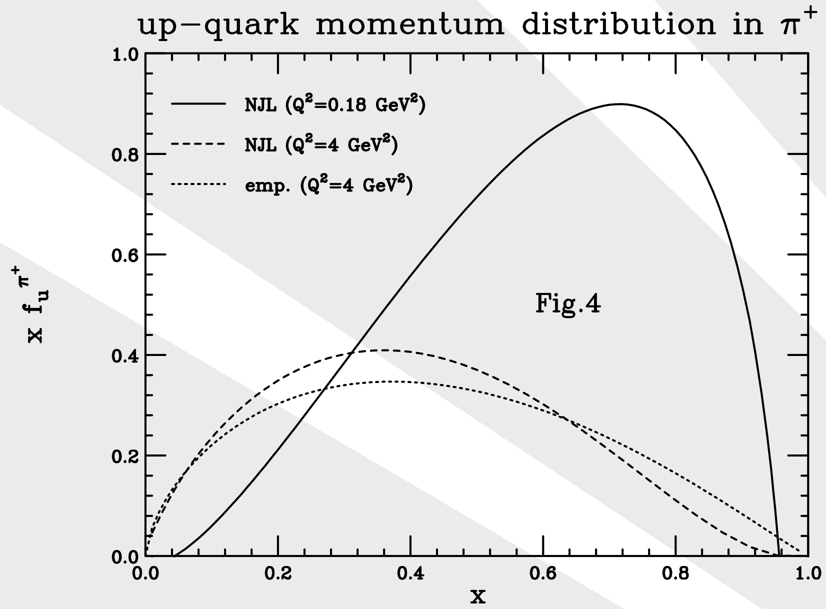
$$S(k) = \frac{\not{k}+M}{k^2-M^2+i\epsilon} = \frac{k^-\gamma^+ + k^+\gamma^- - \mathbf{k}_\perp\cdot\gamma_\perp}{2k^+}\left(\frac{\Theta(k^+)}{k^- - e_k + i\epsilon} + \frac{\Theta(-k^+)}{k^- - e_k - i\epsilon}\right)$$

where $e_k = (\mathbf{k}_\perp^2 + M^2)/2k^+$ and $\mathbf{k}_\perp = (k^1, k^2)$. The result is

$$f_u^{\pi^+}(x) = 6g^2\int \frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\mathbf{k}_\perp^2 + M^2}{\left[\mathbf{k}_\perp^2 + M^2 - m_\pi^2 x(1-x)\right]^2}$$

In this simple valence quark picture of π^+ we have $f_{\bar{d}}^{\pi^+}(x) = f_u^{\pi^+}(x)$.
Experimental information comes from the **DIS structure function**
$F_2^{\pi^+}(x) = x\left(\sum_q e_q^2 f_q^{\pi^+}(x,Q^2) + \sum_{\bar{q}} e_{\bar{q}}^2 f_{\bar{q}}^{\pi^+}(x,Q^2)\right)$, where $q = u, d, \ldots$.
Concerning the Q^2 dependence, see later comments.

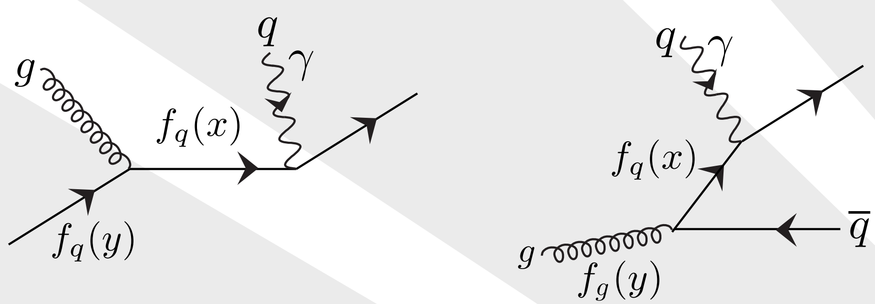
Results for u distribution in π^+



- solid line: NJL result on previous slide, using the invariant mass cut-off scheme (see Notes!)
- dashed line: Q^2 **evolution** up to 4 GeV^2, assigning a low energy scale $Q_0^2 = 0.18$ GeV^2 to the solid (NJL) line
- empirical distribution at $Q^2 = 4$ GeV^2

Notes on the Q^2 evolution

- In NJL model, there are no gluons. But in QCD, a quark can give momentum to a gluon, and the gluon to sea quarks, etc.



- Probability of gluon radiation depends on the "**resolution scale**" (Q^2) in DIS: Probed with higher resolution, more quark momentum appears to be carried by gluons.
- This Q^2 dependence is calculable in **perturbative QCD**, if we know $f_q(x)$ at a low resolution scale (Q_0^2), where we can assume that we have only quarks. This value Q_0^2 defines the energy scale of the NJL model, and is treated as a parameter here.

Comments on the figures

- Fig.1: Solid lines: Dyson-Schwinger parametrizations, see: *A. Holl et al, Phys. Rev.* **C 71** *(2005), p. 065204; Eqs. (63), (64)*. The results do not depend much on the parameter ω if $0.3 < \omega < 0.5$. We show the cases $\omega = 0.4$ (case I) and $\omega = 0.5$ (case II). (For other investigations on the infrared enhancement, see: *M.S. Bhagwat et al, Phys. Rev.* **C 68** *(2003), 015203*; *C.S. Fischer et al, Phys. Rev.* **D 67** *(2003), 094020*.) 1-loop perturbation theory (dotted line): $\alpha_s(k) = \frac{4\pi}{\beta_0}\frac{1}{\ln(k^2/\Lambda^2_{\rm QCD})}$, $\beta_0 = 25/3$, ($N_f = 4$), $\Lambda_{\rm QCD} = 0.234$ GeV.

- Fig.2: Here 3-momentum cut-off is used: $|\mathbf{k}| < \Lambda$ with Λ=0.59 GeV (to reproduce pion decay constant). Chiral symmetry breaking possible for $G_\pi > \pi^2/(6\Lambda^2)$. The case $G_\pi = 6.92$ GeV^{-2} corresponds to quark masses $m = 6.0$ MeV, $M = 400$ MeV.

- Fig.3: Here the proper-time cut-off is used: $\Lambda_{\rm UV} = 0.64$ GeV, $\Lambda_{\rm IR} = 0.2$ GeV. The constituent quark mass is $M = 0.4$ GeV. The calculation follows closely that for the nucleon form factors in: *T. Horikawa et al, Nucl. Phys.* **A 762** *(2005), p. 102*, where the corrections from pion cloud and vector mesons are discussed in detail. Measurements of F_π at low Q^2 are done by scattering pions off the electrons in liquid hydrogen, and by the reaction $p(e, e'\pi^+)n$ (pion electroproduction) at higher Q^2 (at JLab), see: *V. Tadevosyan et al; Phys. Rev.* **C75** *(2007), p. 055205.*

- Fig.4: See *W. Bentz et al, Nucl. Phys.* **A 651** *(1999), p. 143; Fig. 4.* Here the constituent quark mass $M = 0.3$ GeV, and the "invariant mass cut-off" (or "Lepage-Brodsky cut-off") is used ($\Lambda = 1.47$ GeV in the figure), which is essentially equivalent to the 3-momentum cut-off scheme with $\Lambda = 0.67$ GeV. The computer code for the Q^2 evolution is taken from: *M. Miyama, S. Kumano, Comp. Phys. Commun.* **94** *(1996), p. 185.* (We use the next-to-leading-order (NLO) evolution with $\Lambda_{\rm QCD} = 0.25$ GeV.) The empirical quark distribution in the pion is taken from: *P.J. Sutton et al, Phys. Rev.* **D 45** *(1992) 2349*. It is extracted from inclusive Drell-Yan pair production: $\pi^\pm N \to \mu^+\mu^- X$, which mainly arises from the annihilation of a quark in the nucleon with an antiquark in the pion.
[For a good introduction to deep inelastic scattering and Q^2 evolution, see: *R.L. Jaffe, 1985 Los Alamos School on Relativistic Dynamics and Quark Nuclear Physics*, ed. M.B. Johnson and A. Pickleseimer (Wiley, new York, 1985).]